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The Path to Fusion: A Computational Scientist's Journey

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- William Tang

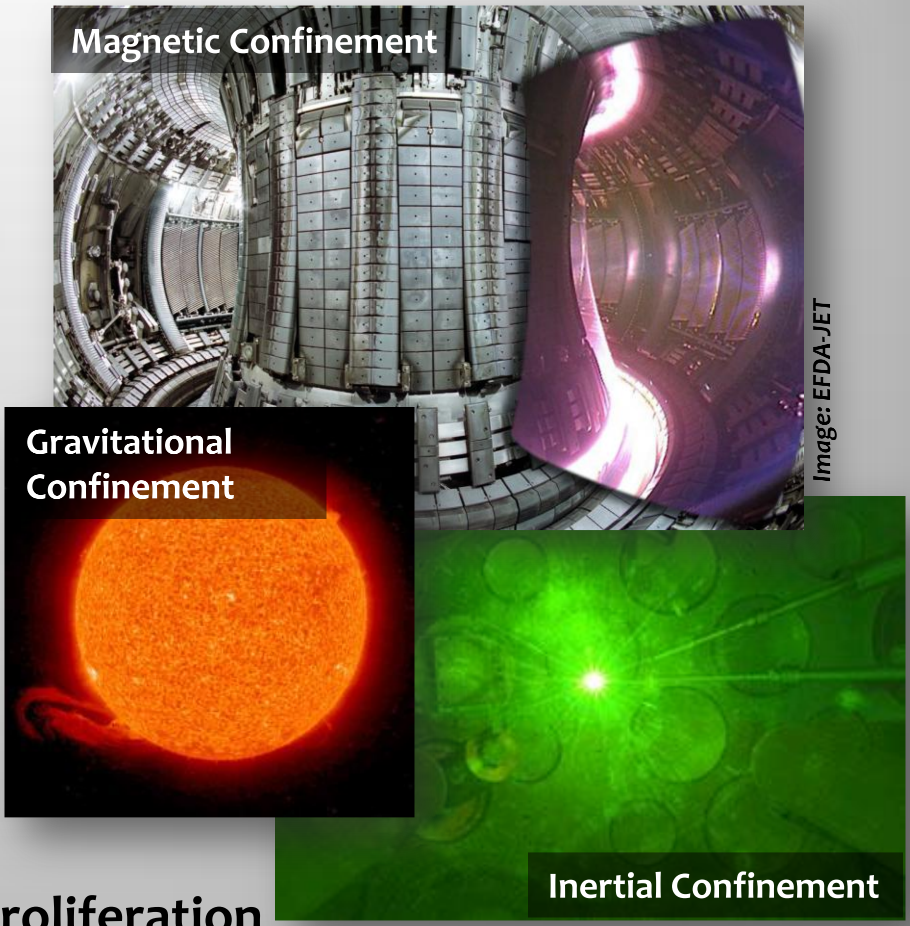
"Hard work is rewarding. Taking credit for other people's hard work is rewarding and faster." – *Scott Adams*

Disclaimers

- The opinions expressed herein, though undoubtedly correct, are entirely my own and are not necessarily those of LLNL, DOE, Krell, or my wife.
- Keep out of reach of children.
- Your mileage may vary.
- Void where prohibited.

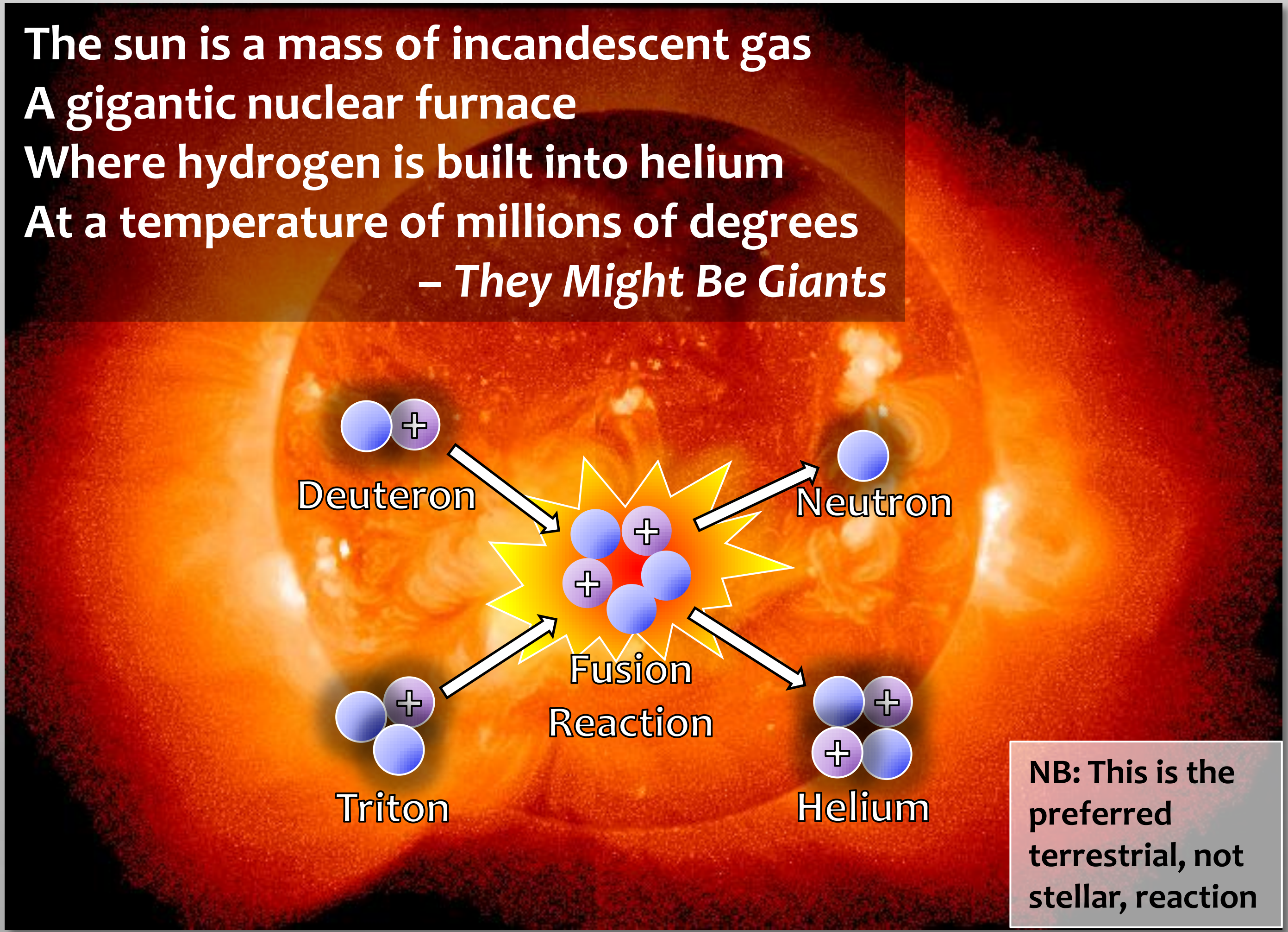
Why fusion?

- Operational advantages
  - Abundant fuel
  - Less severe accident potential
  - Steady source
- Environmental advantages
  - Short-lived radioactivity
  - Low carbon footprint
  - Low land usage
- Security advantages
  - Less attractive terrorist target
  - Low risk of nuclear materials proliferation



Fusion: The Power of Stars

The sun is a mass of incandescent gas
A gigantic nuclear furnace
Where hydrogen is built into helium
At a temperature of millions of degrees
– *They Might Be Giants*

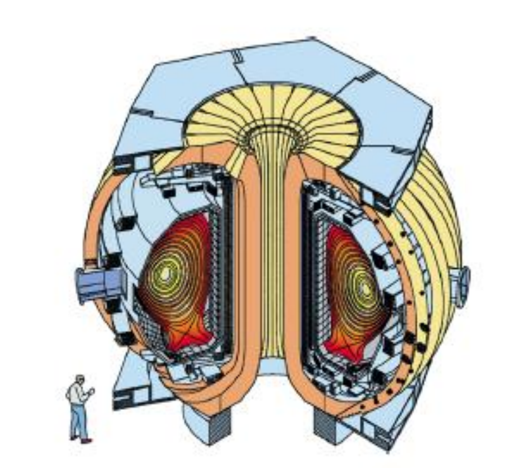


NB: This is the preferred terrestrial, not stellar, reaction

Of course, we can't build a sun on earth, but there are other ways…

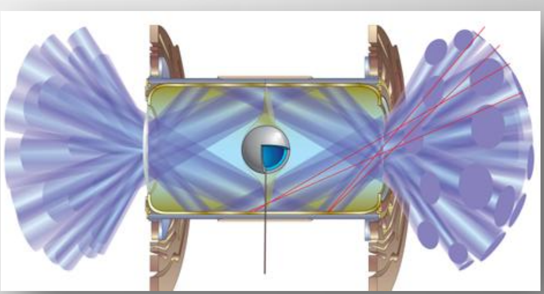
Two of the main approaches to fusion

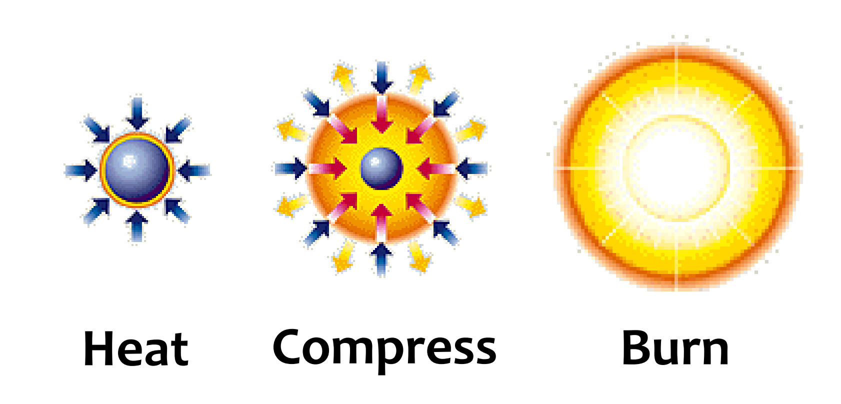
Magnetic Confinement



Toroidal magnetic field confines Ohmically and RF-heated plasma until ignition

Inertial Confinement



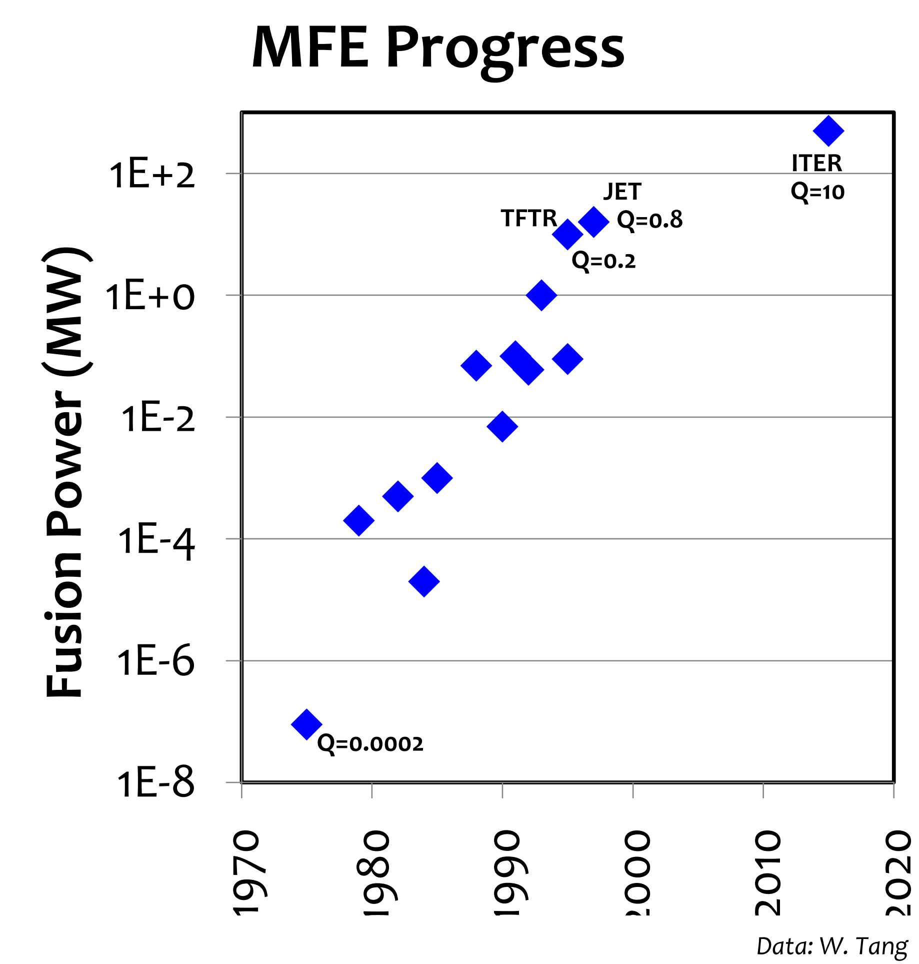


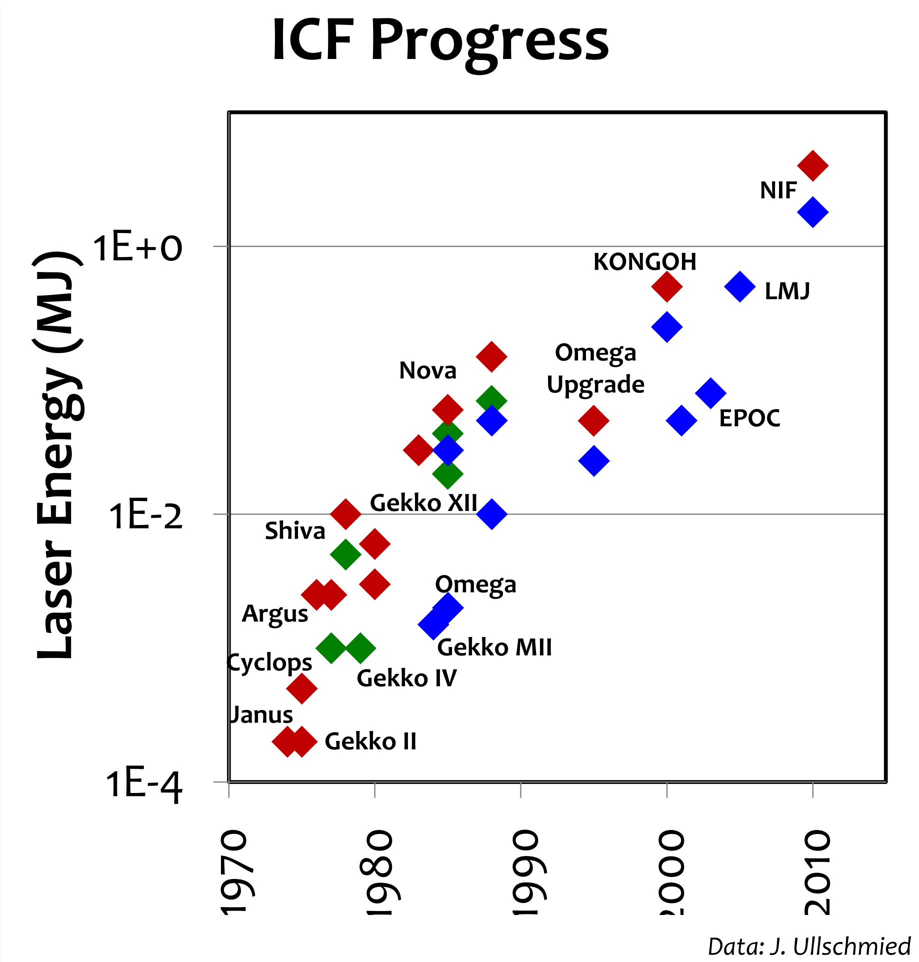
Rapid ionization and blow-off of fuel pellet causes pellet implosion and ignition

There are other methods , such as Z-pinch, inertial electrostatic,…

Fusion is only 20 years away… *and it always will be!*

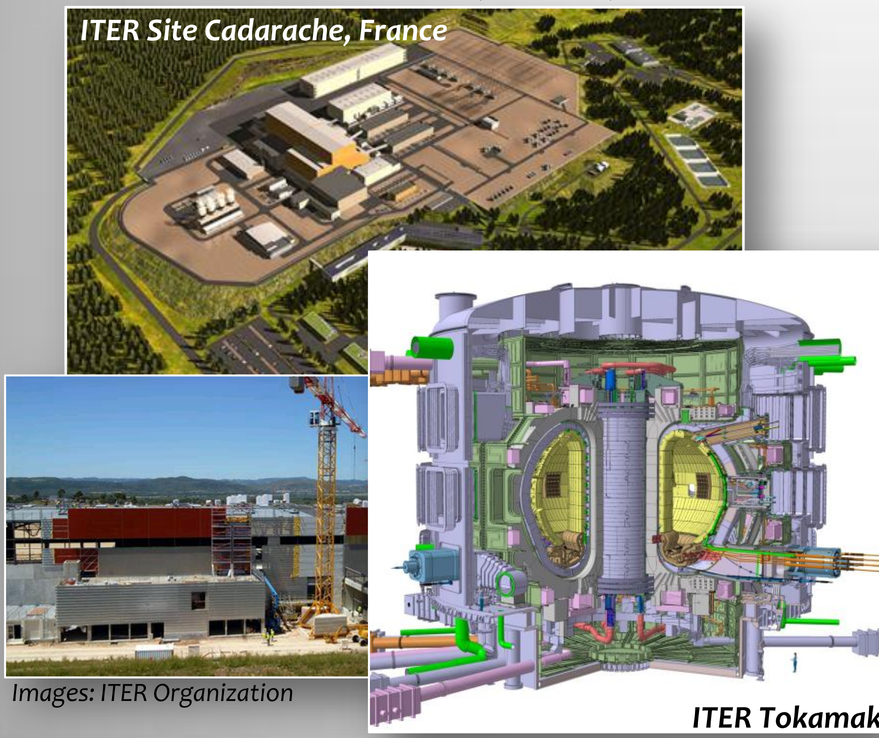
Cute… but this ignores continuing gains in fusion devices





Two major devices coming online this decade

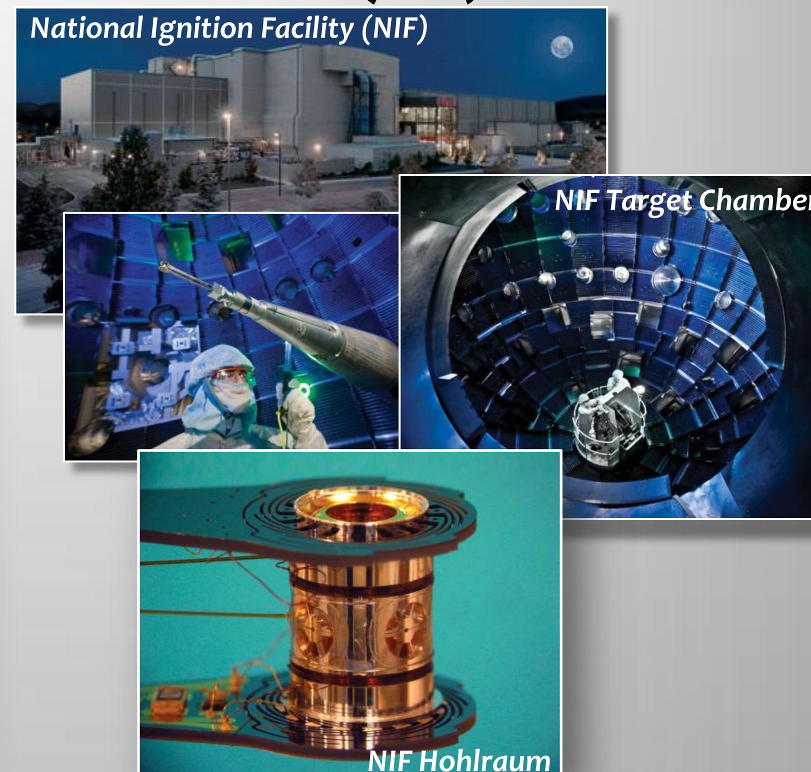
ITER (MFE)



Images: ITER Organization

- 840 m³ plasma volume
- 50 MW input
- ~100's of seconds of burn
- Expected 10x gain (2015)

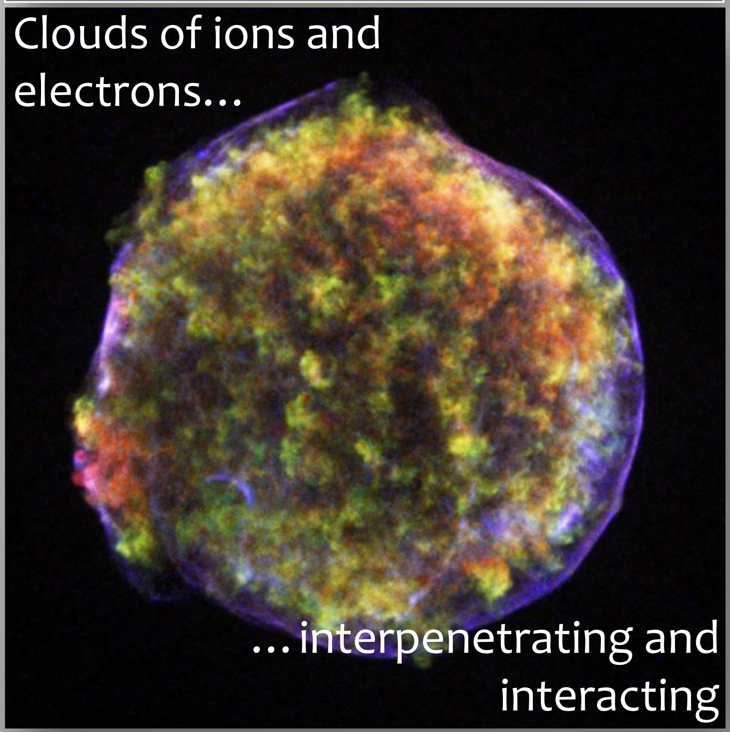
NIF (ICF)



- 192 laser beams
- 2 MJ (500TW) input
- ~1 ns pulse
- Expected 10-20x gain (2013)

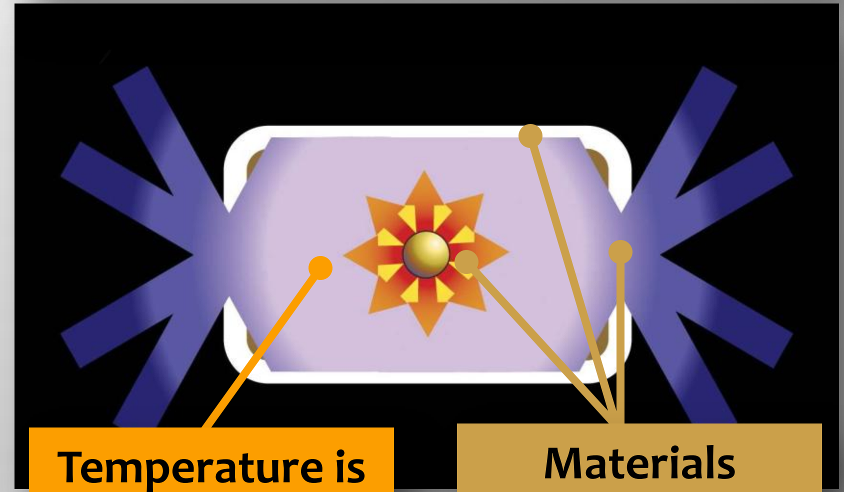
Fusion requires understanding materials at extreme temperatures: plasmas

Plasma is an ionized gas



Tycho's Supernova Remnant, NASA

Hohlraum fills with plasma



Temperature is millions of degrees

Materials ablate and ionize

Plasma motion generates and plasma responds to electromagnetic fields

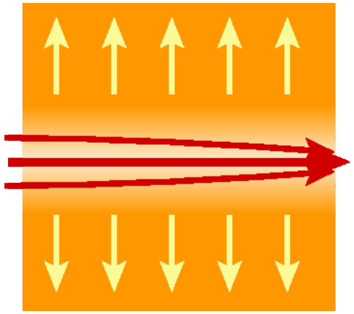
Plasma physics is key to successful fusion

For example, consider laser plasma interactions:

Laser energy is absorbed by plasma

Inverse Brehmsstrahlung

Plasma coupling exchanges energy between crossing laser beams



Lasers “dig” channels through plasma and self-focuses

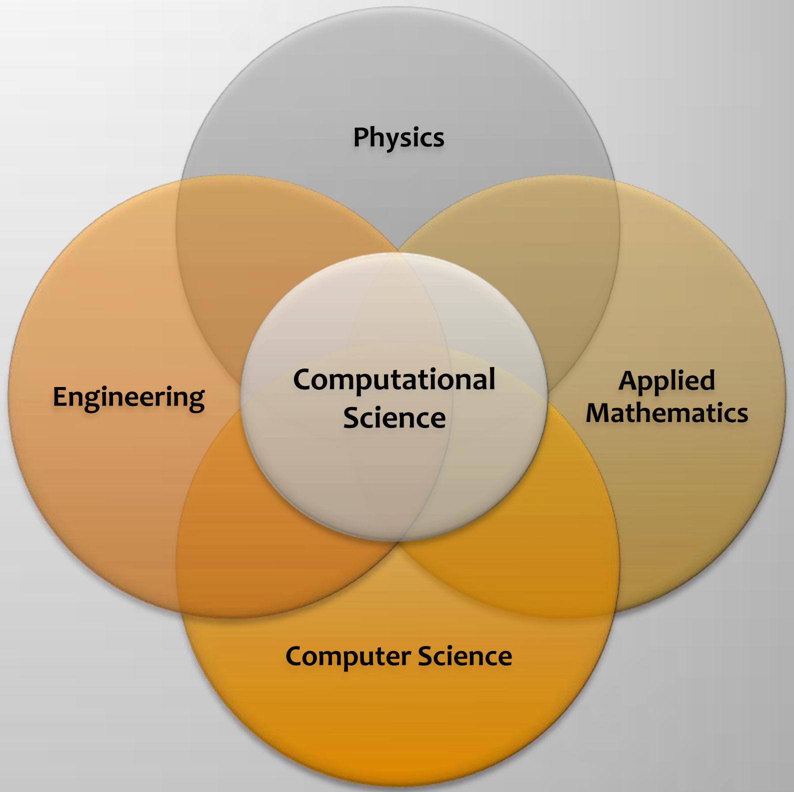
Filamentation and Beam Spray

Laser-driven resonant plasma waves scatter laser light

Stimulated Raman Scattering (SRS)
Stimulated Brillouin Scattering (SBS)

"What are you?"

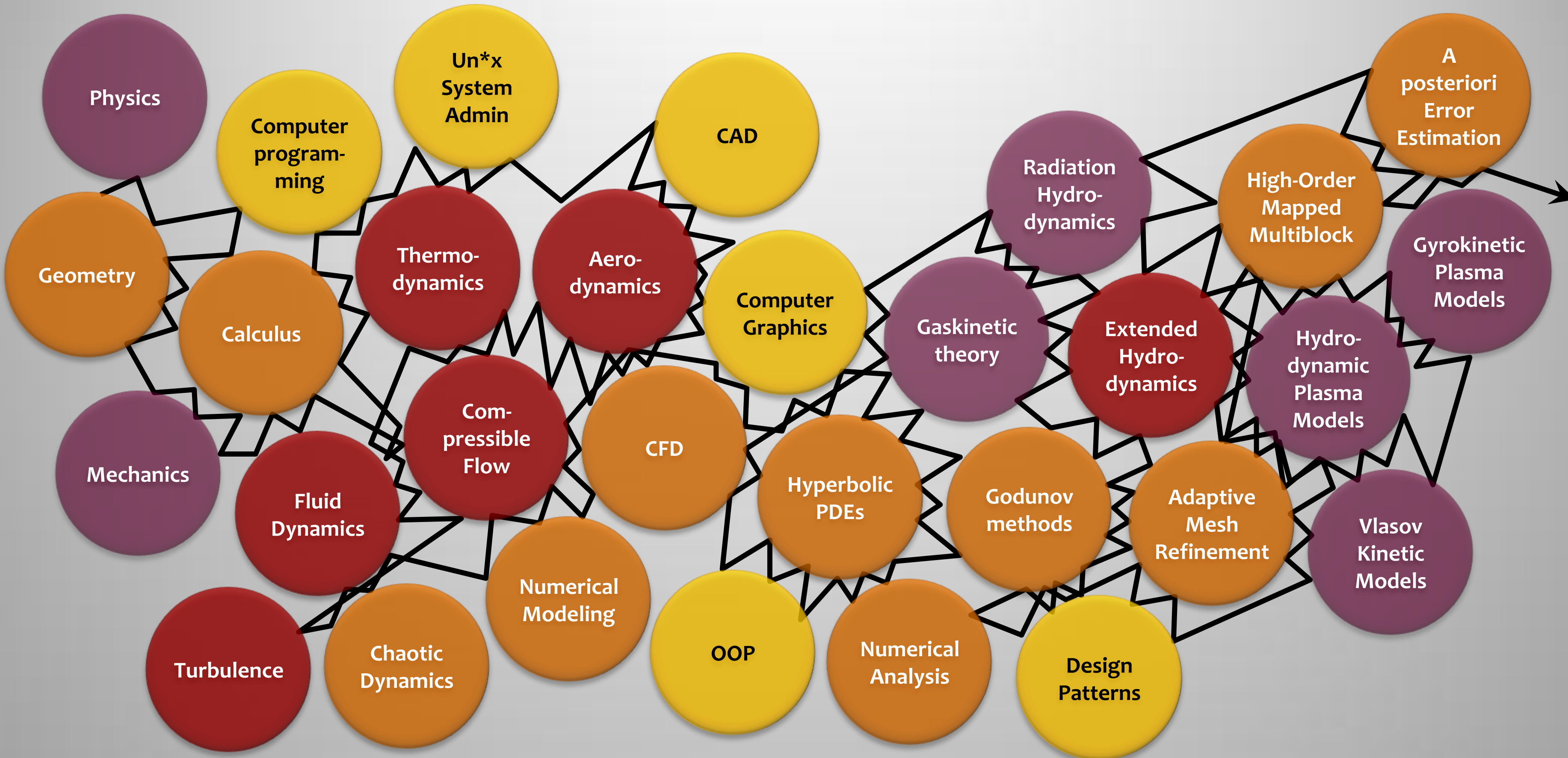
- **A talk including the last slide prompted this question from Ed Moses, Principal Associate Director NIF & Photon Science**
  - He knew I was in the Computation Directorate
  - My comfort with both the computational and physical issues surprised him



Of course, computational science lives at the overlap of many traditional domains

IANAPP*

** I Am Not A Plasma Physicist*



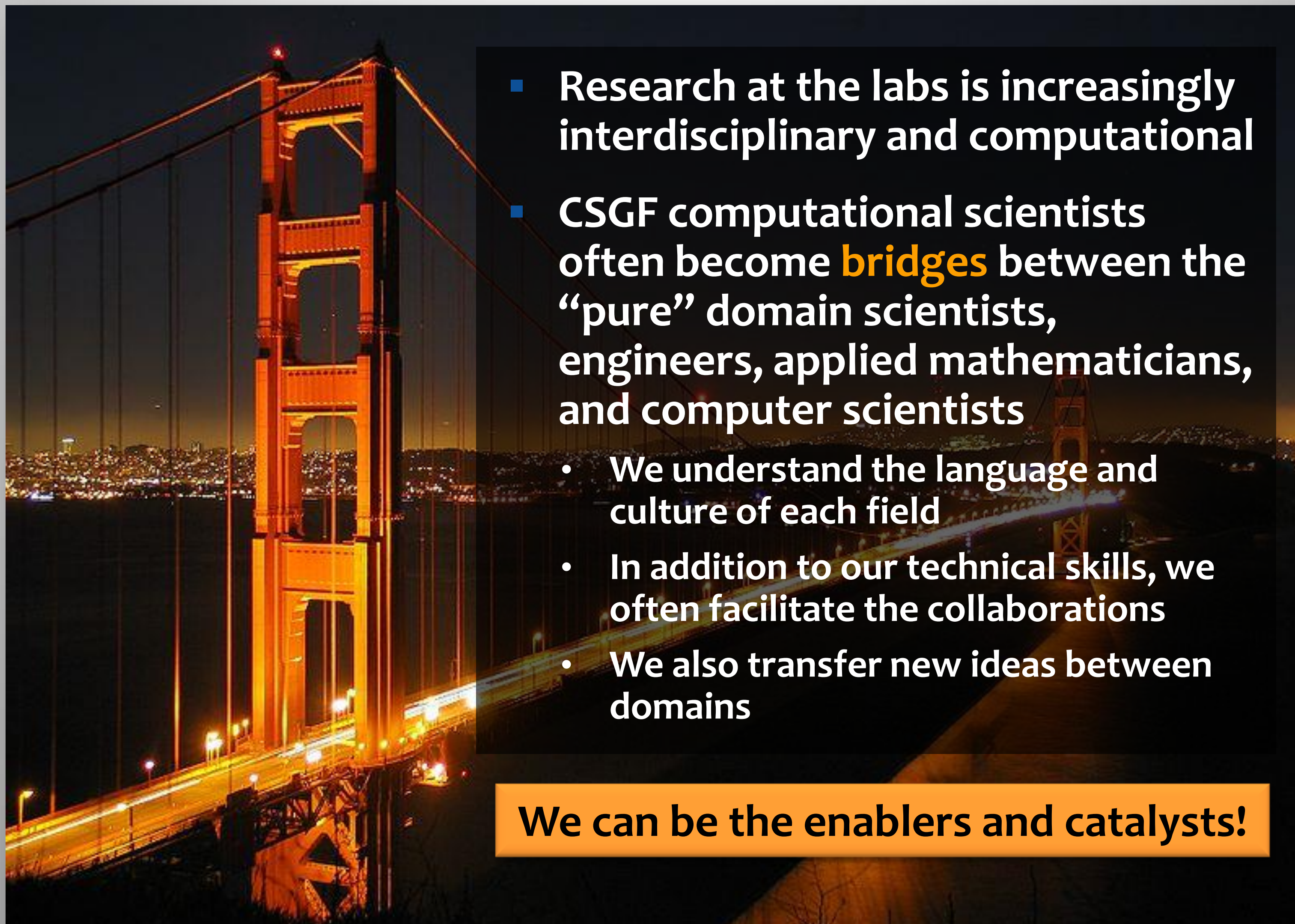
"I seldom end up where I wanted to go, but almost always end up where I need to be." – *Douglas Adams*

CSGF tends to create Chimeras

- Challenges
  - Computational science as a discipline does not have a unified community
  - Interdisciplinary fields often do not fit into the academic hierarchy
- Advantages
  - Endless opportunities to work on a wide array of problems
  - Important role in multidisciplinary teams


Pearson Scott Foresman

Embrace the opportunity to lead!



- Research at the labs is increasingly interdisciplinary and computational
- CSGF computational scientists often become bridges between the “pure” domain scientists, engineers, applied mathematicians, and computer scientists
  - We understand the language and culture of each field
  - In addition to our technical skills, we often facilitate the collaborations
  - We also transfer new ideas between domains

We can be the enablers and catalysts!

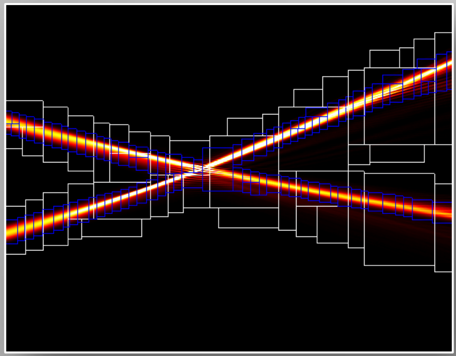
Photo: Daniel Schwen

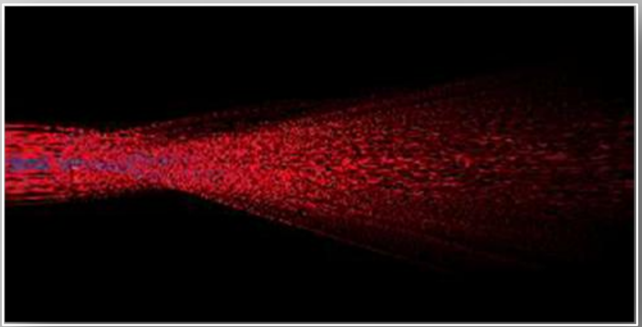
Act I: Plays well with others

In which I describe my early collaborations in plasma physics and the important lessons learned…

A practicum at Los Alamos led to a postdoc at Livermore…

- **Involvement began with the Adaptive Laser Plasma Simulator (ALPS)**
  - Adaptive Mesh Refinement (AMR) applied to Laser Plasma Interaction (LPI)
  - Used modern FAC, multigrid, and high-resolution Godunov algorithms
- **Moved on to support the development of the pF3d LPI code**
  - Developed by and for plasma physicists
  - Goal: a predictive LPI capability for National Ignition Facility (NIF)





How to use our ALPS knowledge to improve pF3d?

An approximate continuum model forms the basis of hydrodynamic-scale LPI simulation

Nonlinear hydrodynamic system:

[MASS]
$$\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0$$

[MOMENTUM]
$$\partial_t (\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v} + p\mathbf{I}) = -\alpha n_e \nabla |\tilde{A}|^2$$

[ION PRESSURE]
$$3/2 \left[\partial_t p_i + \nabla \cdot (p_i \mathbf{v})\right] + p_i \nabla \cdot \mathbf{v} = 0$$

[ELEC. PRESSURE]
$$3/2 \left[\partial_t p_e + \nabla \cdot (p_e \mathbf{v})\right] + p_e \nabla \cdot \mathbf{v} = -n_e \nabla \cdot \mathbf{q} + \beta \nu_a |\tilde{A}|^2$$

$$\left[\begin{array}{rl} p_{e|i} &= n_{e|i} T_{e|i} \\ n_e &= Z n_i \\ \rho &= m_i n_i \\ p &= p_i + p_e \end{array}\right]$$

Paraxial light equation:

[APPROXIMATE FACTORIZATION OF WAVE EQUATION]
$$\left(\partial_t + c\eta_0 \partial_z - \frac{i c \eta_0 \nabla_\perp^2}{k\eta_0 + \sqrt{k^2 \eta_0^2 + \nabla_\perp^2}} + \nu_a\right) \tilde{A} = -i\mu \; \left[n_e - \langle n_e \rangle_{x,y}\right] \tilde{A},$$

[LIGHT VECTOR POTENTIAL]

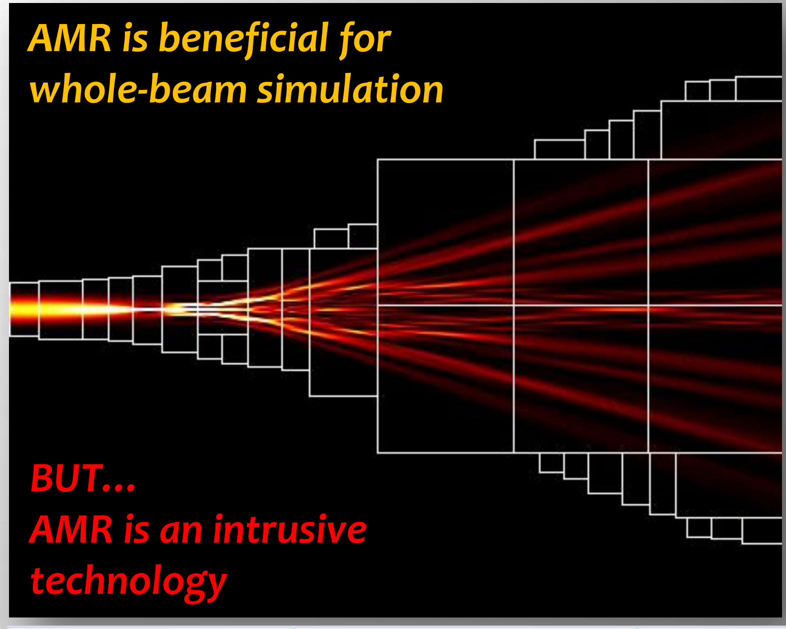
[AVG. REFRACTIVE INDEX]

$$\left[\begin{array}{c} A(\mathbf{x}, t) = \frac{1}{2} \tilde{A}(\mathbf{x}, t) \exp\left(-i\omega t + ik \int_{z_0}^{z} \eta_0(\zeta, t) d\zeta\right) \; + \; \text{c.c.} \\ \eta_0(z, t) = \sqrt{1 - \langle n_e \rangle_{x,y} / n_c} \end{array}\right]$$

The ALPS project demonstrated that advanced numerical methods could benefit LPI simulation

- **2D and 3D filamentation model**
  - Nonlinear fluid plasma model
    - — High-resolution Godunov method
    - — Unsplit Corner Transport Upwind
  - Steady-state paraxial laser model
    - — Energy-conserving Crank-Nicholson
    - — Fast Adaptive Composite (FAC) method
    - — Multigrid solver for paraxial sweep
- **Block-structured Adaptive Mesh Refinement**
  - — Local time stepping
  - — SAMRIA C++ AMR library
  - — Parallelization using MPI

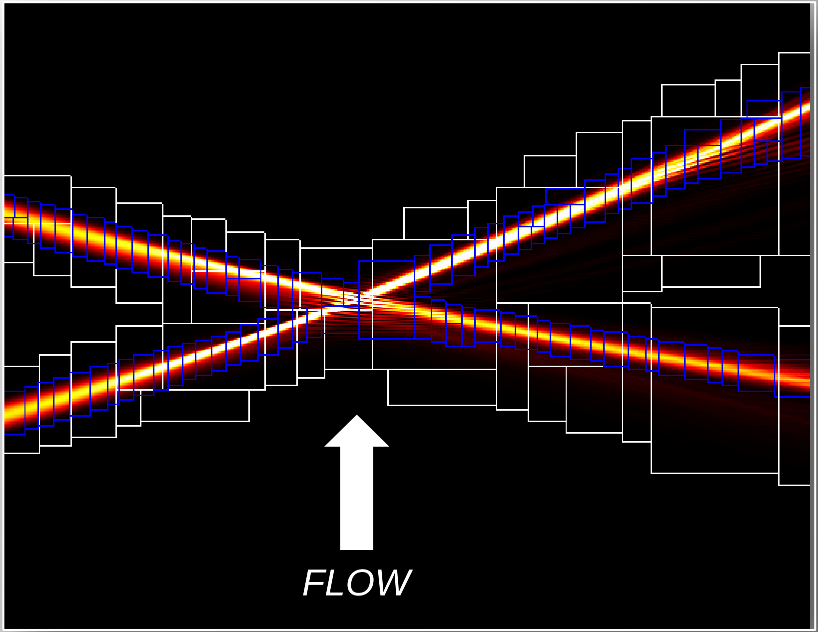
[Dorr, M. R. *et al.* (2002) *J. Comput. Phys.* **177** 233-63]



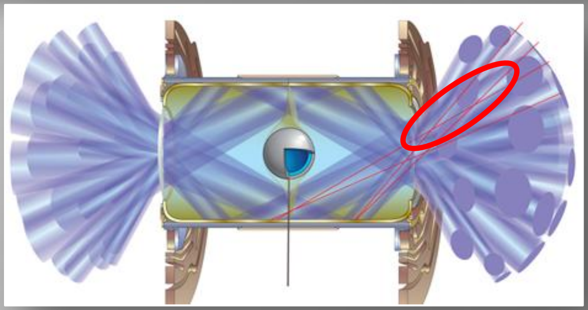
2D AMR Efficiency	CPU Hours		Speed-Up
	Adaptive	Uniform	
Laser Integrator	2.01	3.40	1.70
Plasma Integrator	8.45	29.40	3.48
Other	0.05	0.17	3.09
Total Run Time	10.51	32.97	3.14

ALPS was used for an investigation into of cross-beam energy transfer in support of NIF

- Crossed beams in expanding plasma
- Lasers couple through stimulated plasma wave (SBS)
- Energy from one beam redirected into other beam



Conclusion: Crossed beam energy transfer can be mitigated using the NIF "two-color" option - the ability to slightly change the frequency of the beam cones

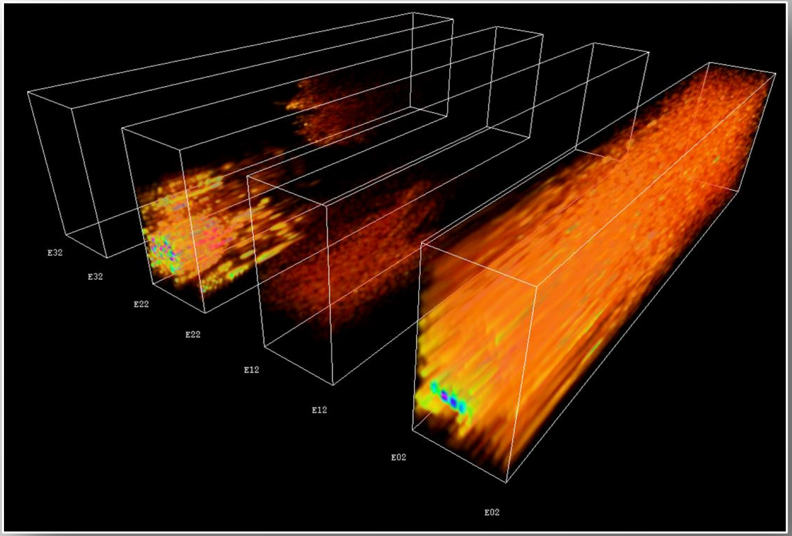


[Kirkwood *et al.* (2002) *Phys. Rev. Let.* **89** 215003]
[Hittinger *et al.* (2005) *J. Comput. Phys.* **209** 695-729]
[Kirkwood *et al.* (2005) *Phys. Plasmas* **12** 112701]

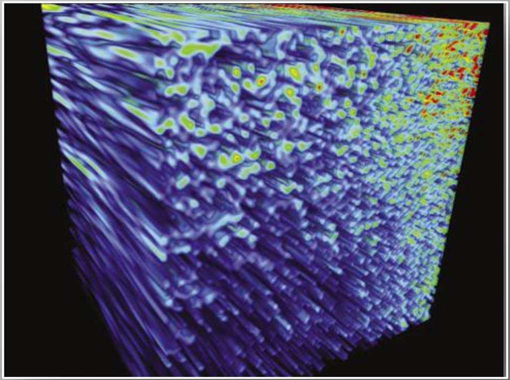
As more capabilities were required, we incorporated new technologies into the NIF LPI code pF3d

pF3d Formulation

- Nonlinear fluid plasma model
  - Arbitrary Lagrangian Eulerian
- Time-resolved paraxial light model
  - Explicit models for SBS & SRS
- Realistic beam profiles
- Other reduced models, e.g.,
  - nonlinear saturation
  - Landau damping



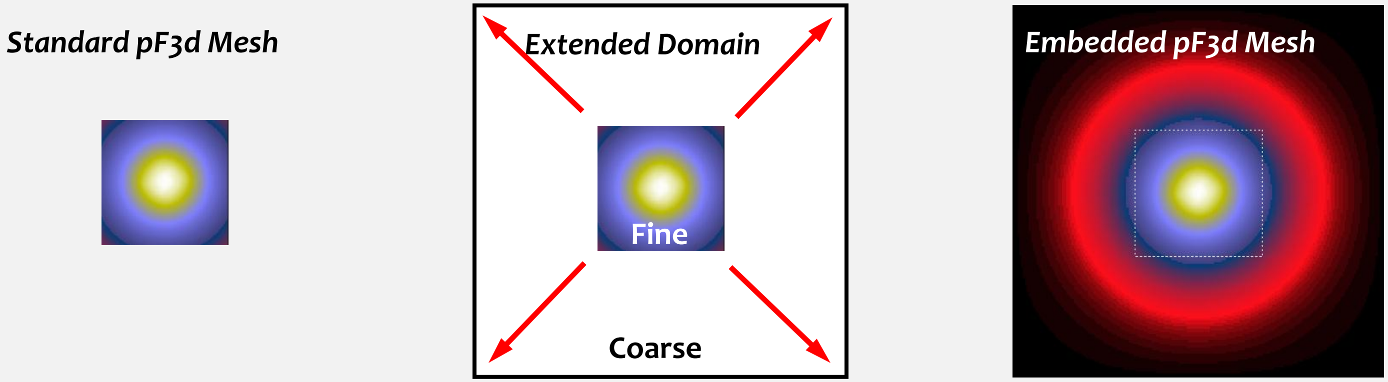
Required Capabilities	New Technology
Large beams with plasma heating	Multilevel multigrid solver
Hohlraum plasmas with different materials	Multifluid Godunov solver



We added a scalable, nonlinear classical electron heat conduction capability to pF3d

- Spitzer-Härm flux-limited local heat-transfer model
- Approximate Newton iteration for nonlinearity
- Geometric multigrid solver for resulting linear systems
  - Red-black Gauss-Seidel smoothing
- Fast-Adaptive-Composite iteration to synchronize coarse and fine grids

A Two-Grid Approach to Model Far-Field Boundary Conditions



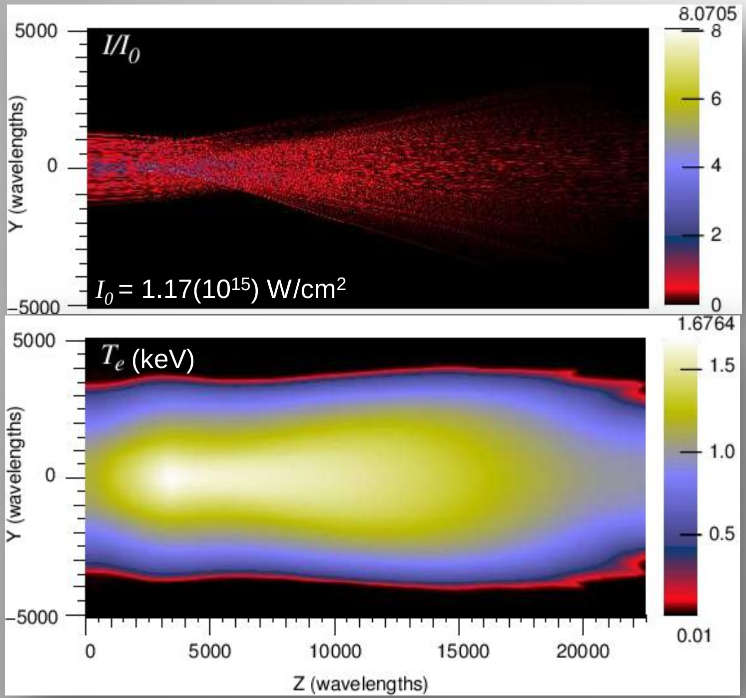
Artificial boundary too close to specify appropriate boundary conditions

Push artificial boundaries away from standard grid

Solve temperature on both grids

[Hittinger, J. A. F. and Dorr, M. R. (2006) *J. Phys. Conf. Series* **46** 422-32]

Simulation of beam propagation experiments in the NIF Early Light (NEL) campaign



Fine grid: 10240λ x 5632λ

Coarse grid: 640 x 704

Coarse domain: 20480λ x 22528λ

240 to 360 hours per run on 64 1.4GHz Itanium2 Processors

[Glenzer, S. H. *et al.* (2007) *Nature Phys.* **3** 716-9]

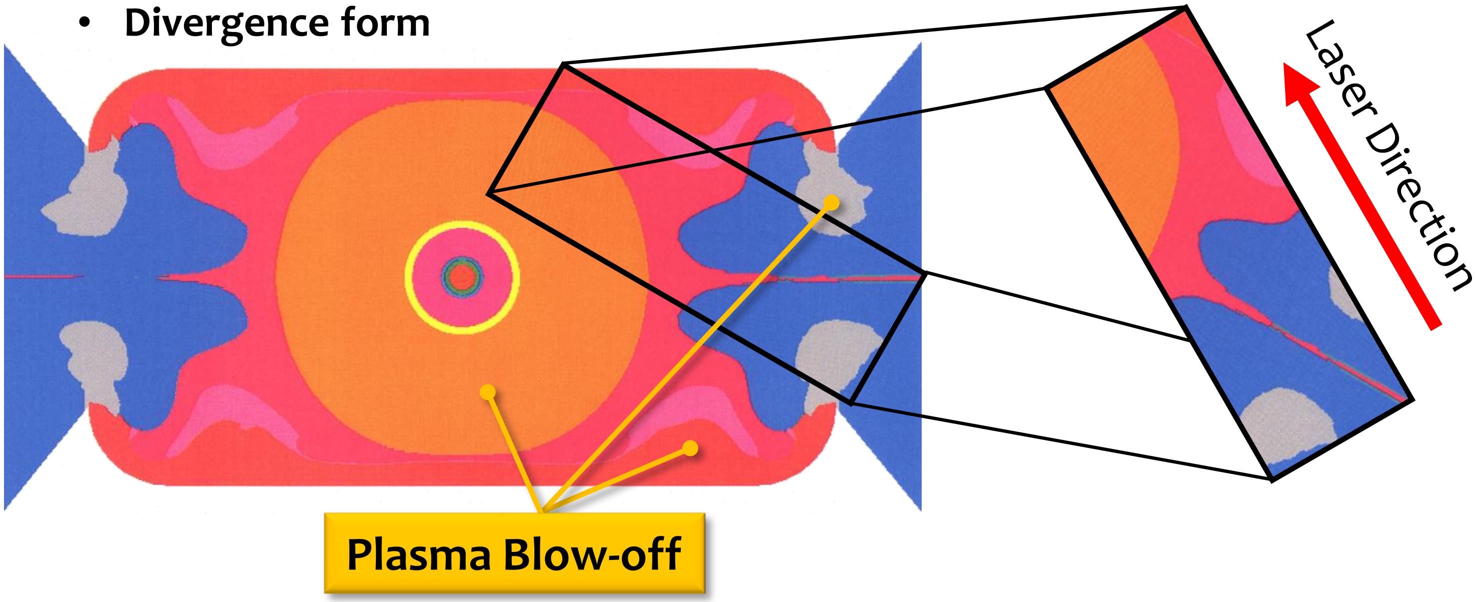
- **Experiments of beam propagation through plasmas at *NIF length and time scales* (7 mm, 3.5 ns)**
- ***Spitzer-Härm heat conduction is critical***
  - **Initially cold plasma absorbs laser energy**
- **Three laser beam conditioning techniques were evaluated**
- **Time for laser to reach far-end of plasma differed by up to 2 ns**
- **3D simulations only feasible for <100ps runs**
- **2D simulations with Spitzer-Härm, in addition to other techniques, *reproduced laser transmission trends***

Filamentation was identified as the cause of stalled propagation

We added a robust multifluid nonlinear hydrodynamic capability in pF3d

Plasma Multifluid Model

- Volume fraction model
  - Additional linearly degenerate waves in hyperbolic subsystem
- Solved with high-resolution Godunov method
  - Eulerian formulation
  - Divergence form

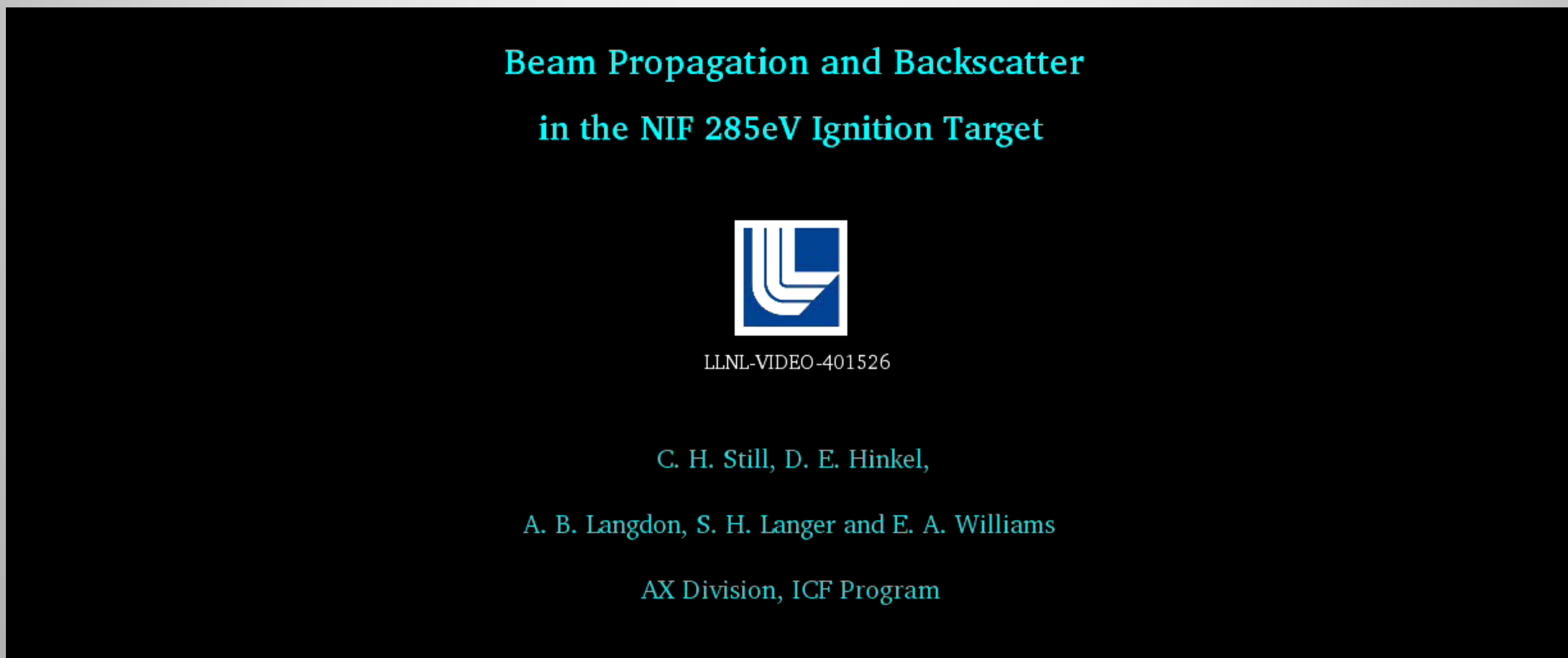


The multifluid solver has become the default hydro solver in pF3D

[Hittinger and Dorr (2006) *J. Phys. Conf. Series* **46** 422-32]

Our new multi-fluid hydro model enabled LPI studies on multi-material hohlraum plasma data

Beam Propagation and Backscatter in the NIF 285 eV Ignition Target



Domain:
- 30° (inner) beam
- LEH to hohlraum wall
- 180 μm x 2.1 mm x 6mm
- 256 x 3072 x 6144 mesh

Compute Time: 10 days on 4096 CPUs of Atlas

Calculation predicts 6% total reflectivity from SBS and SRS

Lead by example

Don’t be dogmatic

- There are few absolutes
  - Methods and models are just tools
  - Depending on the problem, there are *better* choices, but not *correct* choices
- Don’t discount the experience of domain scientists
  - These are smart people who bring important insights

Get your hands dirty

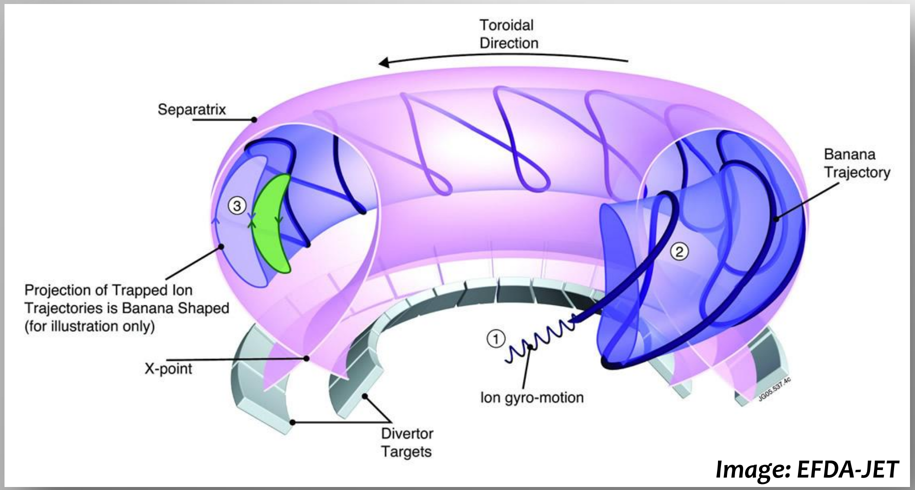
- Domain scientists want a commitment to their problem
- Generate capital: roll up your sleeves and work on their code
  - Hold your nose if you must
  - Demonstrate the effectiveness of your methods
  - Success provides opportunities for further improvement

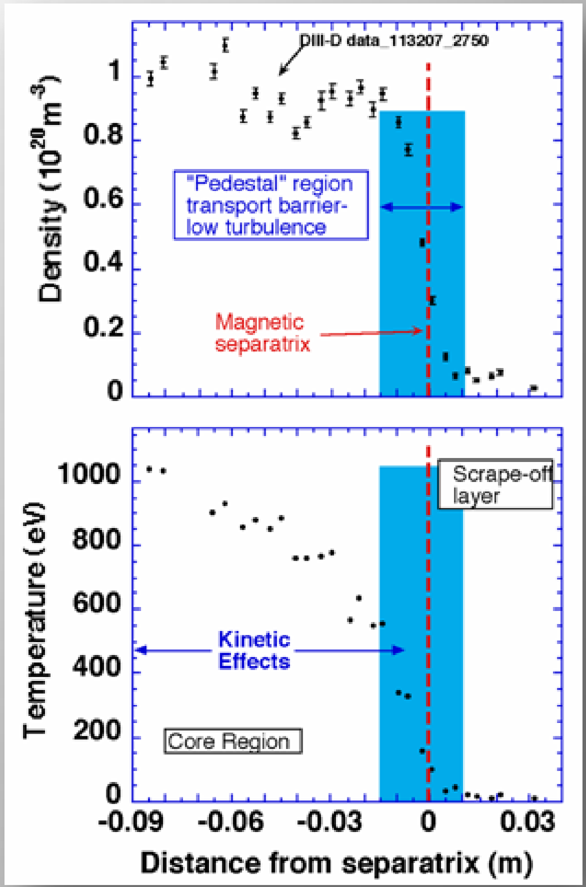
“You do not lead by hitting people over the head. That's assault, not leadership. ” - Dwight D. Eisenhower

Act II: Adjusts easily to new situations

In which I describe later collaborations in plasma physics and reflect on success in the DOE laboratories…

Based on our earlier work, we were asked to collaborate on fusion reactor edge plasma simulation





The edge pedestal density (ne) and temperature (Te) profiles near the edge of an H-mode discharge in the DIII-D tokamak. The horizontal scale is distance from nominal boundary of the plasma at R= 2.34m [from G.D. Porter, et al., Phys. Plasmas 7 (7), 3663 (Sept. 2000)].

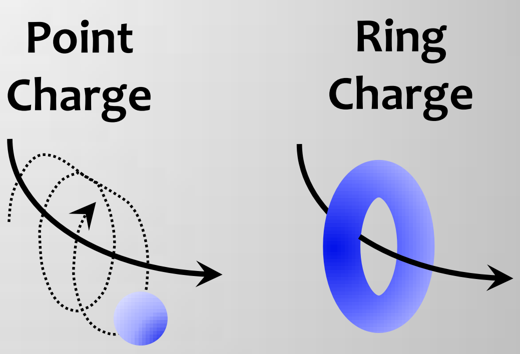
- In high-performance (H-mode) discharges, a steep-gradient region (the pedestal) develops
  - Pedestal becomes a transport barrier
  - Kinetic models are required to model the pedestal evolution
- Extension to the plasma edge of continuum gyrokinetic models requires new algorithms to satisfy demanding requirements

Gyrokinetic models are well established in plasma physics, but raise new challenges for edge simulation

- Kinetic models describe the evolution of distribution functions phase space

$$f(x, v, t) : \mathbb{R}^D \times \mathbb{R}^D \times [0, \infty) \rightarrow \mathbb{R}^+$$

- Gyrokinetic models decouple the gyromotion
  - Average gyro-motion is like propagating ring charges instead of point charges
  - Reduces 6D phase space to 5D
  - Removes a fast time scale
- Used to simulate core turbulence for many years
- The plasma edge differs from the core
  - Can not use perturbation formulation
  - Strong, rapidly varying density and temperature in pedestal yields overlapping time scales
  - More complicated geometry



Numerical approach	Pros	Cons
Particle	Easy to parallelize, potentially efficient phase space resolution	Statistical noise, $N^{-1/2}$ error dependence
Continuum	No noise, potentially better error control	Requires a phase space grid

Our target model has been a gyrokinetic Vlasov-Poisson system in magnetic edge geometry

Vlasov:

$$\frac{\partial}{\partial_t}(B_\parallel^* f) + \nabla_R(\dot{\mathbf{R}} B_\parallel^* f) + \frac{\partial}{\partial_{v_\parallel}}(\dot{v}_\parallel B_\parallel^* f) = 0$$

describes the evolution of a distribution function

$$f \equiv f(t, \mathbf{R}, v_\parallel)$$

in gyrocenter phase space coordinates

Gyrokinetic Poisson (in long wavelength limit):

$$\nabla \cdot \left(\left[\lambda_D^2 \mathbf{I} + \lambda_L^2 \sum_i \frac{Z_i \bar{n}_i}{m_i \Omega_i^2} (\mathbf{I} - \mathbf{b}\mathbf{b}^T) \right] \nabla \Phi \right) = n_e - \sum_i Z_i \bar{n}_i$$

gyrophase-dependent ion density

gyro-averaged ion density

$\mathbf{B}$	**Equilibrium magnetic field**
Φ	**Equilibrium potential**
La	**Larmor number (normalized gyroradius)**

$$\dot{\mathbf{R}} \equiv \frac{v_\parallel}{B_\parallel^*}\mathbf{B}^* + \frac{La}{Z B_\parallel^*}\mathbf{b} \times \mathbf{G} \qquad \dot{v}_\parallel \equiv -\frac{1}{m B_\parallel^*}\mathbf{B}^* \cdot \mathbf{G}$$

$$\mathbf{B}^* \equiv \mathbf{B} + La \frac{m v_\parallel}{Z} \nabla_R \times \mathbf{b} \qquad B_\parallel^* = \mathbf{b} \cdot \mathbf{B}^*$$

$$\mathbf{G} \equiv Z \nabla_R \Phi + \frac{\mu}{2} \nabla_R |\mathbf{B}| \qquad \mathbf{b} \equiv \mathbf{B}/|\mathbf{B}|$$

We employ a systematic formalism for high-order, mapped-grid finite volume discretizations

Cartesian coordinates:

Spatial domain discretized as a union of rectangular control volumes

$$V_{\mathbf{i}} = \prod_{d=1}^{D} \left[i_d - \frac{h}{2}, i_d + \frac{h}{2} \right]$$

Mapped coordinates:

Smooth mapping from abstract Cartesian coordinates into physical space

$$\mathbf{X} = \mathbf{X}(\boldsymbol{\xi}), \qquad \mathbf{X} : [0,1]^D \to \mathbb{R}^D$$

Fourth-order flux divergence average from fourth-order cell face averages:

$$\int_{\mathbf{X}(V_{\mathbf{i}})} \nabla_{\mathbf{X}} \cdot \mathbf{F} d\mathbf{x} = \sum_{\pm=+,-} \sum_{d=1}^{D} \pm \int_{A_d^{\pm}} \left(\mathbf{N}^T \mathbf{F}\right)_d d\mathbf{A}_{\boldsymbol{\xi}} = h^{D-1} \sum_{\pm=+,-} \sum_{d=1}^{D} \pm F^d_{\mathbf{i}\pm\frac{1}{2}\mathbf{e}^d} + O\left(h^4\right)$$

where

$$F^d_{\mathbf{i}\pm\frac{1}{2}\mathbf{e}^d} = \sum_{s=1}^{D} \langle N_d^s \rangle_{\mathbf{i}\pm\frac{1}{2}\mathbf{e}^d} \langle F^s \rangle_{\mathbf{i}\pm\frac{1}{2}\mathbf{e}^d} + \frac{h^2}{12} \sum_{s=1}^{D} \left(\mathbf{G}_0^{\perp,d} \left(\langle N_d^s \rangle_{\mathbf{i}\pm\frac{1}{2}\mathbf{e}^d} \right) \right) \cdot \left(\mathbf{G}_0^{\perp,d} \left(\langle F^s \rangle_{\mathbf{i}\pm\frac{1}{2}\mathbf{e}^d} \right) \right)$$

$\mathbf{G}_0^{\perp,d} =$ second-order accurate centered difference of $\nabla_{\boldsymbol{\xi}} - \mathbf{e}^d \dfrac{\partial}{\partial \xi_d}$

$$\langle q \rangle_{\mathbf{i}\pm\frac{1}{2}\mathbf{e}^d} \equiv \frac{1}{h^{D-1}} \int_{A_d} q(\boldsymbol{\xi}) d\mathbf{A}_{\boldsymbol{\xi}} + O\left(h^4\right)$$

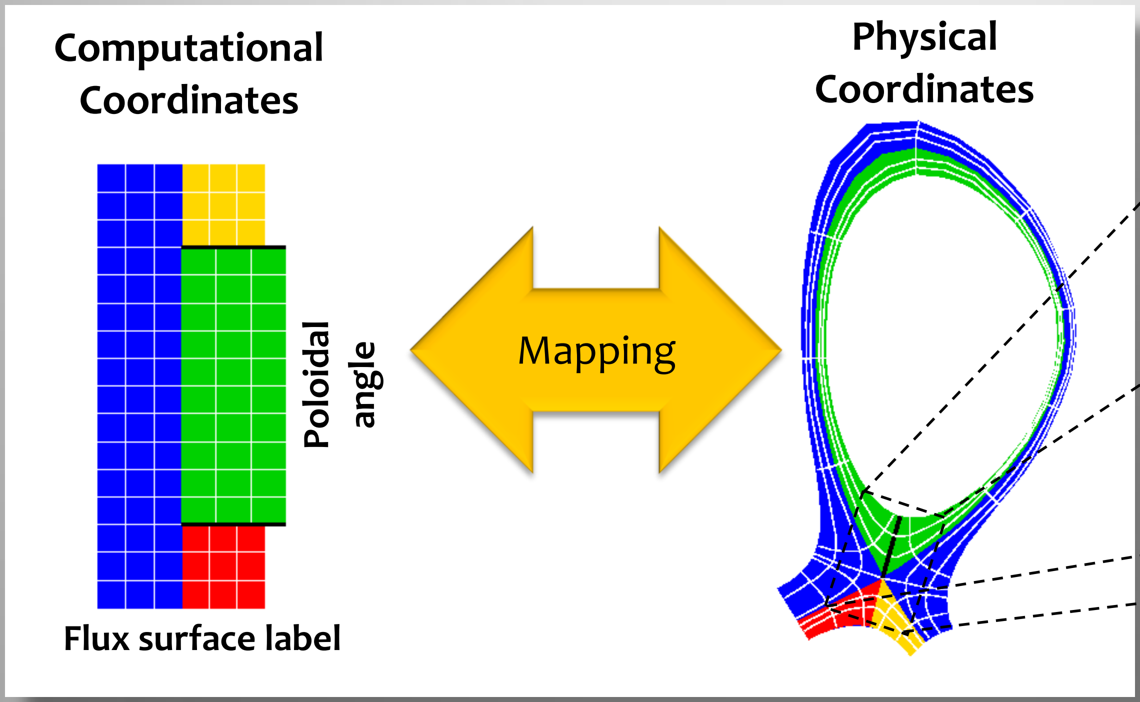
Free streaming is preserved:

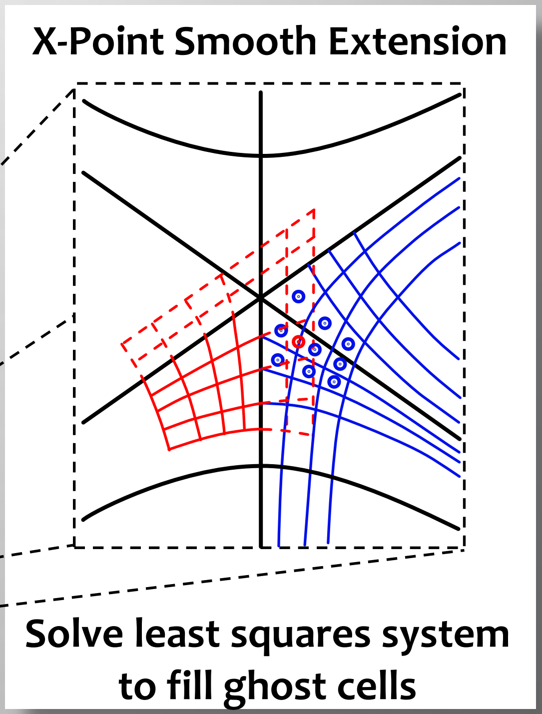
$$\int_{A_d} N_d^s d\mathbf{A}_{\boldsymbol{\xi}} = \sum_{\pm=+,-} \sum_{d' \neq d} \pm \int_{E_d^{\pm}, d'} M^s_{d,d'} d\mathbf{E}_{\boldsymbol{\xi}} \Longrightarrow \int_{\mathbf{X}(V_{\mathbf{i}})} \nabla_{\mathbf{X}} \cdot \mathbf{F} d\mathbf{x} = 0 \text{ for } \mathbf{F} \text{ constant}$$

[Colella, P. et al. (2011) *J. Comput. Phys.* **230** 2952-2976]

Plasma edge geometries can be gridded using mapped multiblock coordinates

- The equilibrium magnetic field determines a mapping from physical to computational coordinates
- Alignment with flux surfaces facilitates treatment of strong anisotropies

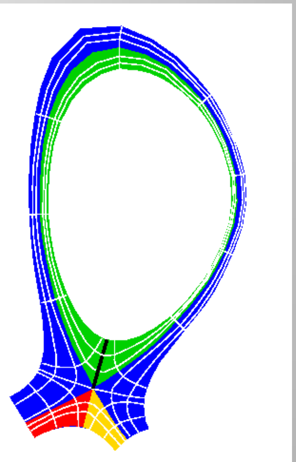




In 2D, a poloidal slice of the plasma edge is mapped to a multiblock, locally rectangular grid

We are developing a new continuum edge code named COGENT (COntinuum Gyrokinetic Edge New Technology)

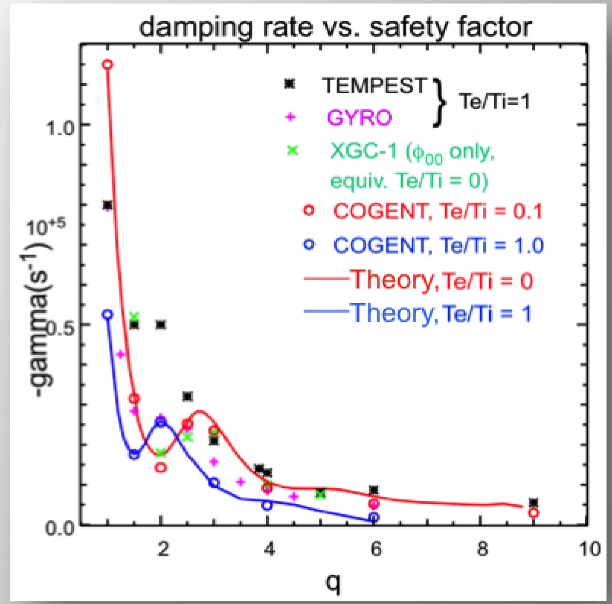
- Successor to TEMPEST prototype code
- Built on Chombo infrastructure (via APDEC)
  - Locally rectangular data containers and communication operations
  - Supports the finite-volume, mapped multiblock formalism
- Parallel in all phase space dimensions, with independent domain decomposition for configuration and phase space
- Component operations involve multiple dimensionalities:

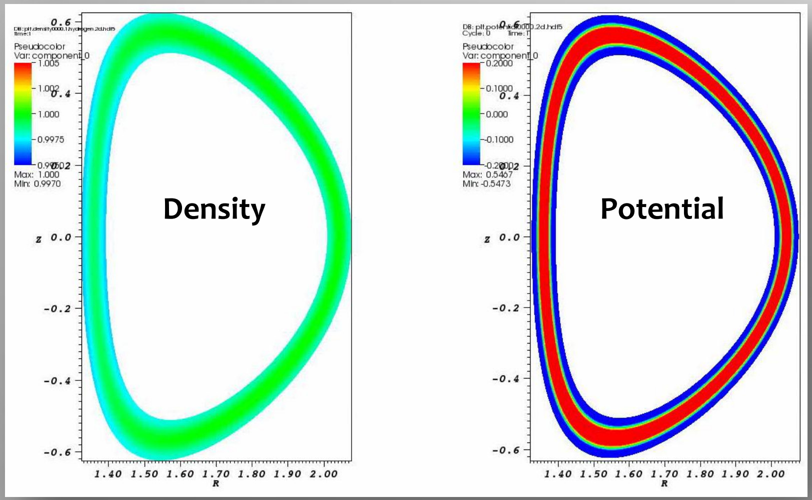


Moment calculation (4D ->2D)	Field solve (2D)	Velocity update (2D ->4D)	Vlasov integration (4D)
$n(t,\mathbf{x}) = \int f(t,\mathbf{x},\mathbf{v})d\mathbf{v}$	$L\Phi = n_i - n_e$ $\mathbf{E} = -\nabla_{\mathbf{x}}\Phi$	$\mathbf{E}, \mathbf{B}, \ldots \rightarrow \mathbf{v}$	$\partial_t f + \nabla \cdot (\mathbf{v} f) = 0$

COGENT predictions of Geodesic Acoustic Mode (GAM) frequencies and damping rates agree well with theory

- Geodesic Acoustic Mode: an eigenmode of the gyrokinetic Vlasov-Poisson system in toroidal geometry
- Kinetic effects cause collisionless damping
- Frequencies and damping rates predicted from a theoretical dispersion analysis



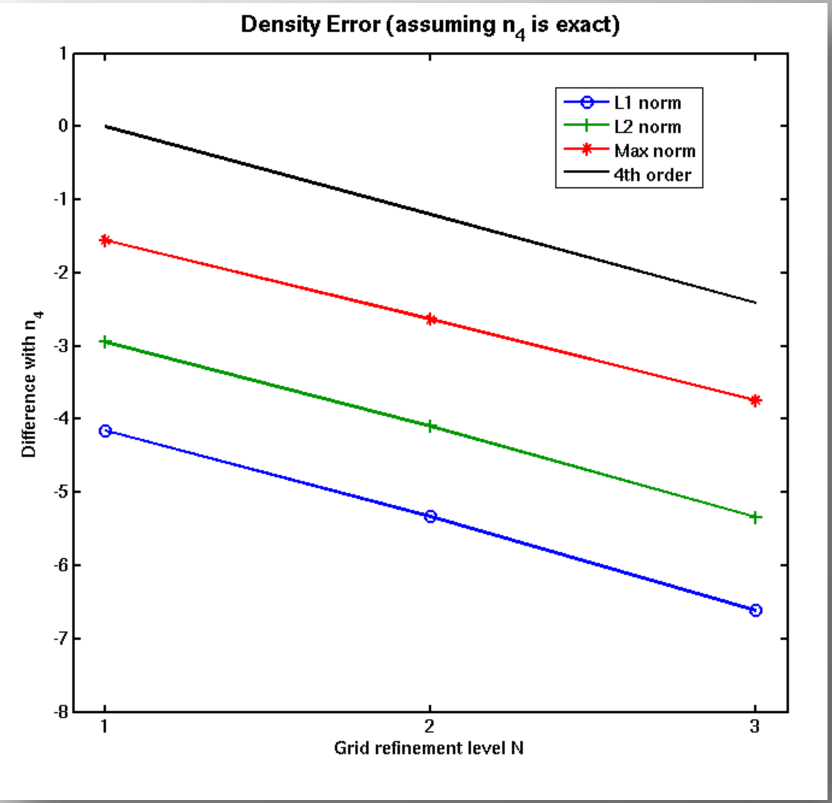


- Tests involved parameter scans of
  - q = field line pitch (“safety factor”)
  - T_e/T_i = electron/ion temperature ratio

[Dorr, M. R. *et al.* (2010) *Proc. SciDAC 2010*]

Comparison of GAM runs at varying phase space resolution displays fourth-order convergence

Grid refinement level (N)	Grid (radial x poloidal x vparallel x mu)	Estimated density error conv. rate (r)	Richardson extrapolated density error (e)
1	8 x 32 x 32 x 8		
2	16 x 64 x 64 x 16		
3	32 x 128 x 128 x 32	3.8 (L1) 3.8 (L2) 4.1 (Max)	3.37×10^{-7} (L1) 6.03×10^{-7} (L2) 1.95×10^{-4} (Max)
4	64 x 256 x 256 x 64	4.2 (L1) 4.1 (L2) 3.6 (Max)	1.40×10^{-8} (L1) 2.95×10^{-7} (L2) 1.69×10^{-5} (Max)



n_i = density at refinement level i

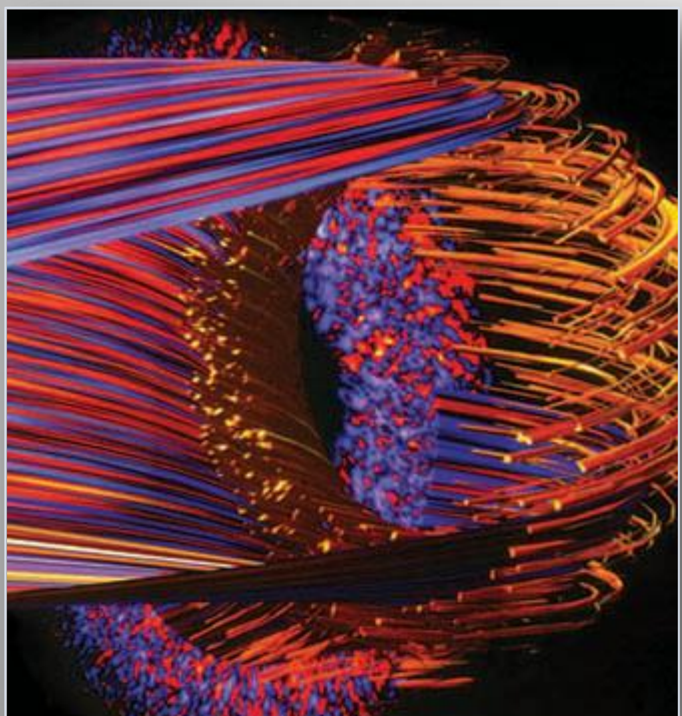
Rate estimate: $r = \log(d_{i+1}/d_i) / \log(2)$

$d_i = || n_{i+1} - n_i ||_{x}$, x = L1, L2, Max

Error estimate: $e = \log(d_i) / (1 + 2^r)$

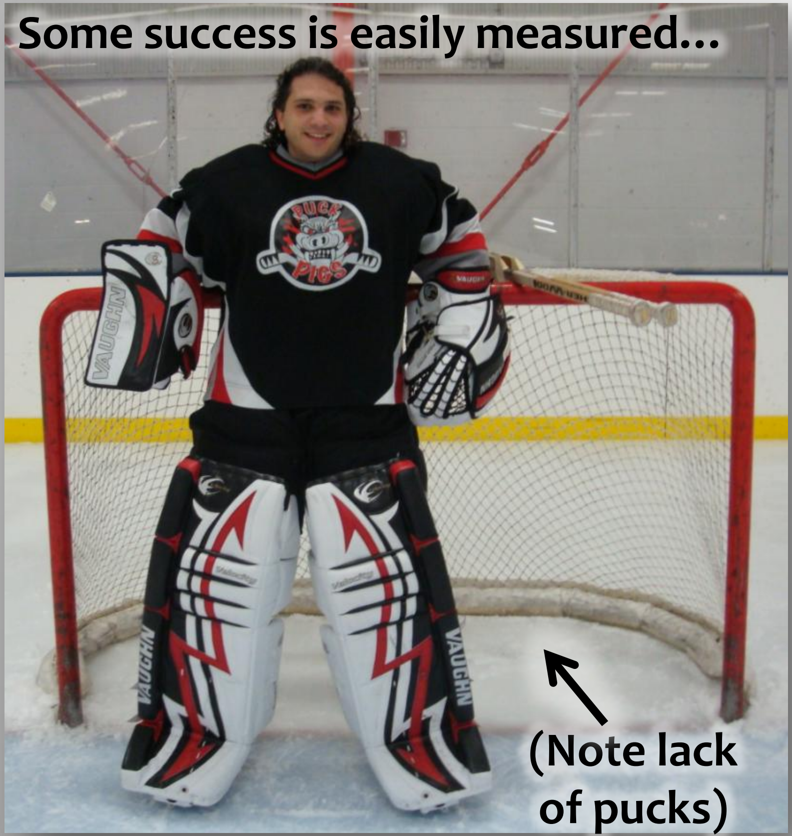
My reputation from plasma simulation and verification efforts within the lab led to work on a national program

FSP FUSION SIMULATION PROGRAM



- FSP is a proposed effort to develop an *integrated, predictive* simulation capability for the MFE community
- We are currently in the final planning phase and will produce a program plan by the end of FY11
- I have lead responsibility for the testing, verification and UQ portions of the plan

What is success?



- Certainly, standard professional metrics are important:
  - Peer-reviewed journal articles
  - Community service (editorial boards, conference organization, etc.)
- More meaningful measure is that others seek out and rely upon your expert opinion
  - This gives you the ability to influence programs and therefore the trajectory of research

"No one ever attains success by simply doing what is required of him." – Charles Kendall Adams

Act III: Assumes responsibility (but runs with scissors)

In which I describe my recent collaborations in plasma physics and reflect on leading a research team…

In 2008, I had the opportunity to lead my own team working to develop new methods for kinetic LPI simulation

Approach

- Apply *high-order, limited Eulerian* (*cf.* semi-Lagrangian) *finite-volume* algorithms to the Vlasov-Maxwell (Poisson) system in high-dimensional phase space

Efficiency

- Parallel adaptive mesh refinement (AMR)
- High-order spatial and temporal discretizations

High-Fidelity

- Positivity preservation
- Control of unphysical oscillations
- Discrete conservation

Restrict consideration to the electrostatic Vlasov-Poisson system for electrons in a field of fixed ions

- State variables:

$$f(x,v,t) : \mathbb{R}^D \times \mathbb{R}^D \times [0,\infty) \to \mathbb{R}^+ \qquad x, v \in \mathbb{R}^D$$
$$\phi(x,t) : \mathbb{R}^D \times [0,\infty) \to \mathbb{R} \qquad t > 0$$

- Electron Vlasov-Poisson system:

$$\partial_t f + \nabla_x \cdot (v f) - \nabla_v \cdot (E f) = 0$$
$$\nabla_x^2 \phi = \int_{-\infty}^{\infty} f \; dv - 1 \qquad E = -\nabla_x \phi$$

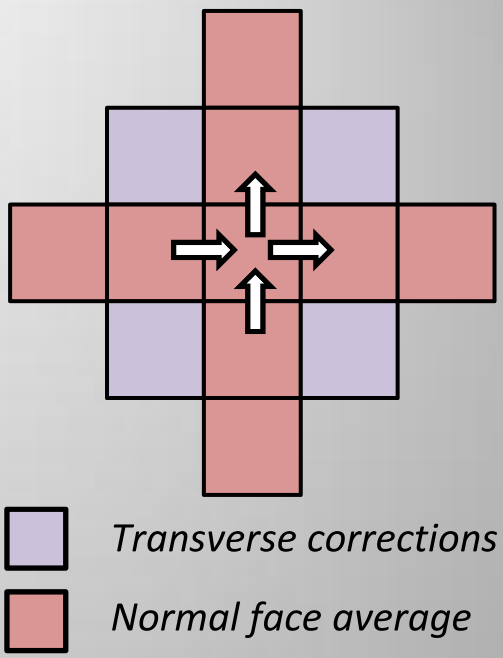
- With initial and boundary conditions:

$$f(x,v,0) = f_0(x,v)$$
$$f(x+P,v,t) = f(x,v,t) \qquad P = \sum_{d=1}^{D} 2 n_d L_d \frac{x_d}{|x|}$$
$$\lim_{|v|\to\infty} f(x,v,t) = 0 \qquad \forall n_d \in \mathbb{Z} \qquad L_d > 0$$

We used a fourth-order method-of-lines discretization based on a finite-volume representation

- Update of cell average:

$$\frac{d\bar{f}_{\mathbf{i}}}{dt} = -\frac{1}{h}\sum_{d=1}^{2D}\left(\langle F\rangle^{d}_{\mathbf{i}+\frac{1}{2}\mathbf{e}^{d}} - \langle F\rangle^{d}_{\mathbf{i}-\frac{1}{2}\mathbf{e}^{d}}\right)$$



- Define phase-space coordinates and velocity:

$$\xi = (x, v) \qquad u = (v, -E)$$

- Face-averaged flux[‡]:

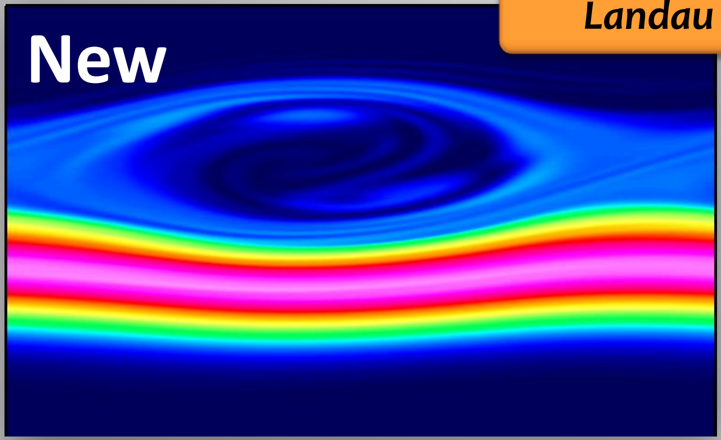
$$\langle F\rangle^{d}_{\mathbf{i}\pm\frac{1}{2}\mathbf{e}^{d}} = \left(\langle u^{d}\rangle\langle f\rangle + \frac{h^2}{12}\sum_{d'\neq d}\frac{\partial u^{d}}{\partial \xi_{d'}}\frac{\partial f}{\partial \xi_{d'}}\right)_{\mathbf{i}\pm\frac{1}{2}\mathbf{e}^{d}} + O\left(h^4\right)$$

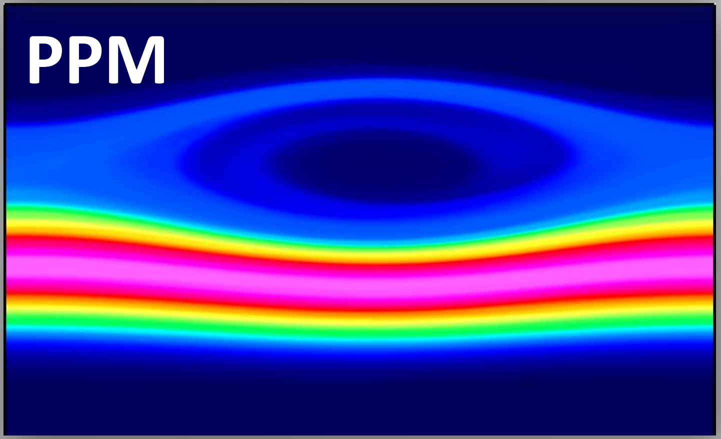
- Standard explicit four-stage, fourth-order Runge-Kutta in time

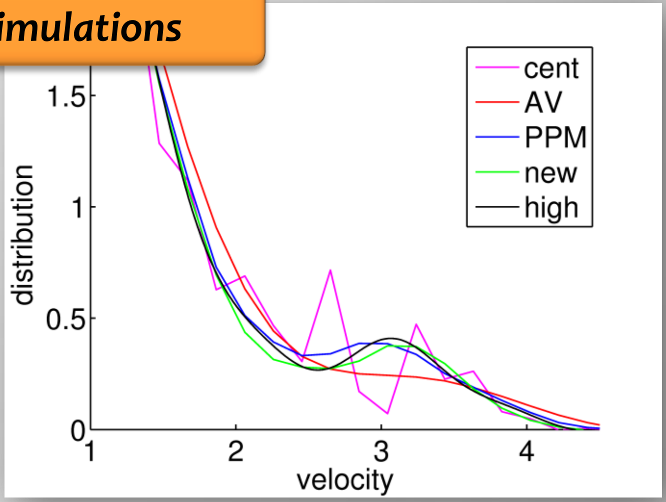
‡Colella, Dorr, Hittinger, & Martin, *J Comput Phys* 230 (2011) 2952-2976

New scheme is less dissipative than genuine shock capturing schemes like the Piecewise Parabolic Method (PPM)

Landau damping simulations





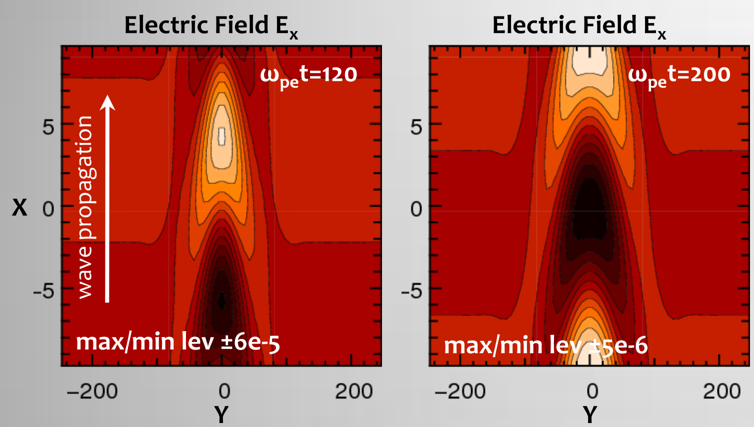


Positivity of the distribution function can be enforced directly with an application of the Zalesak multidimensional FCT scheme

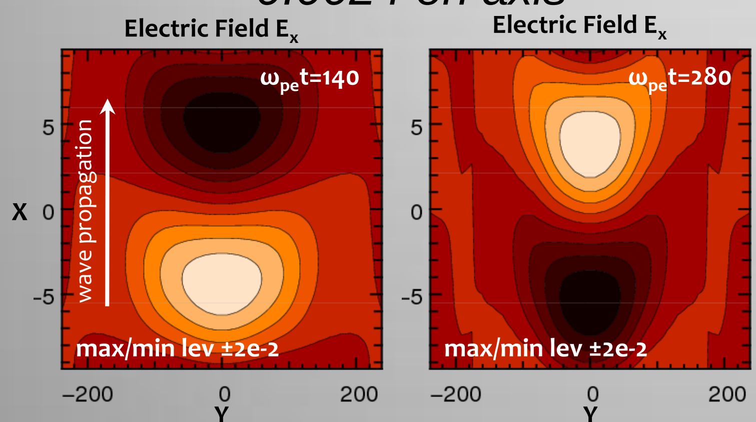
- For under-resolved scales, the upwind-biasing adds dissipation to suppress oscillations, but strict monotonicity is not preserved
- At the same resolution, the new scheme has higher fidelity

[Banks and Hittinger (2010) *IEEE Trans. Plasma Sci.* **38** 2198-207]

Simulations of driven, finite-width EPWs show linear and nonlinear bowing in agreement with theory



$\Delta y/\lambda_{De}$=12, Peak $e\phi/T_e$ = 0.0024 on axis



$\Delta y/\lambda_{De}$=90, Peak $e\phi/T_e$ = 0.075 on axis

Linear Wavefront Bowing

$$C_{\rm L}(t) = -\frac{3}{2}\left(\frac{\lambda_{De}}{\sigma(t)\Delta y}\right)^2 \frac{\omega_{pe}^2}{\omega_{k_0}} t.$$

- Results from linear dispersion of EPWs
- Phase velocity is slower off-axis leading to ***negative wavefront curvature***
- Dominates for ***small amplitudes waves and early times***

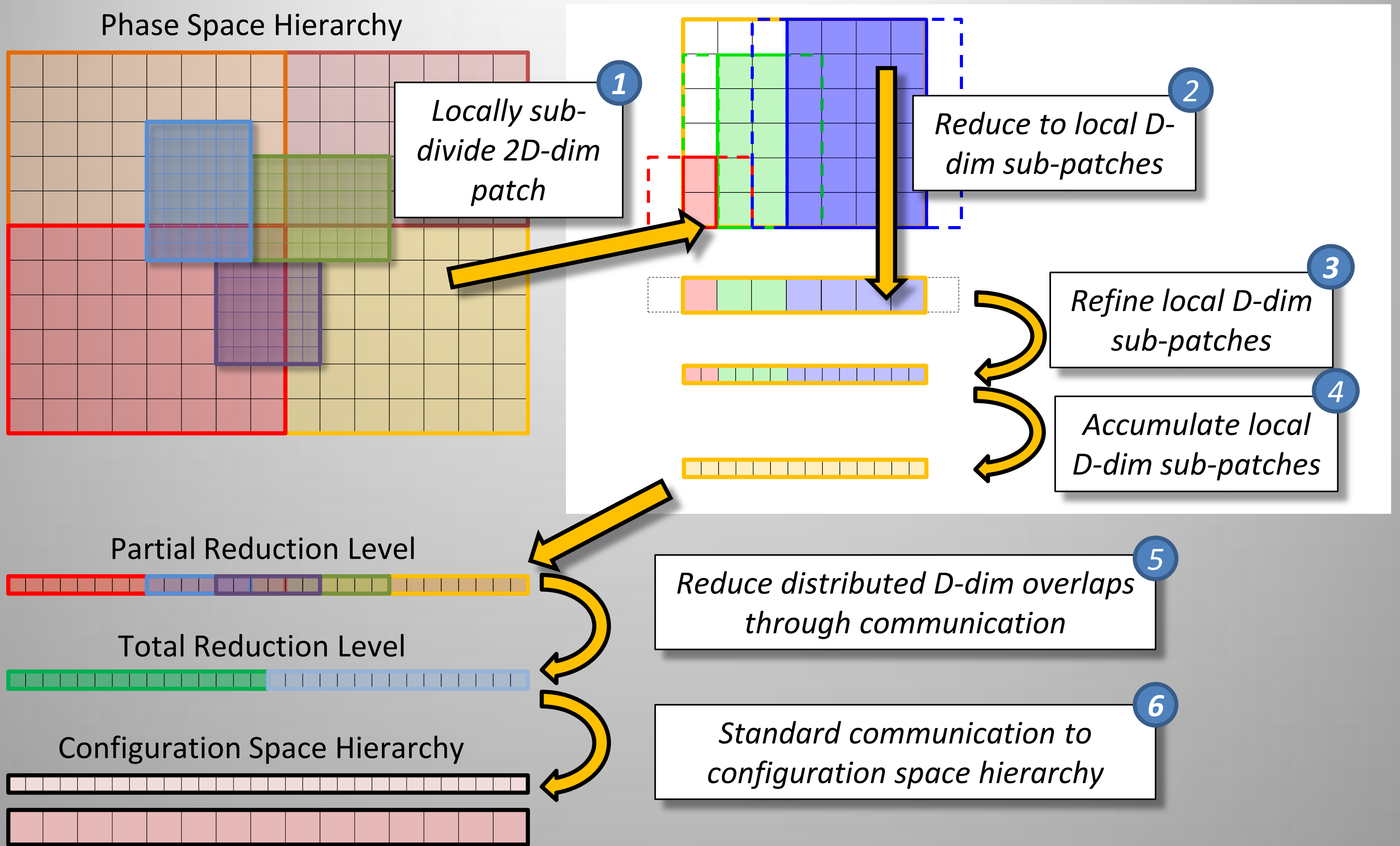
Nonlinear Wavefront Bowing

$$C_{\rm NL}(t) = -\frac{1}{4}\frac{\delta\omega_{{\rm NL},0}}{\omega_{pe}}\omega_{pe} t,$$

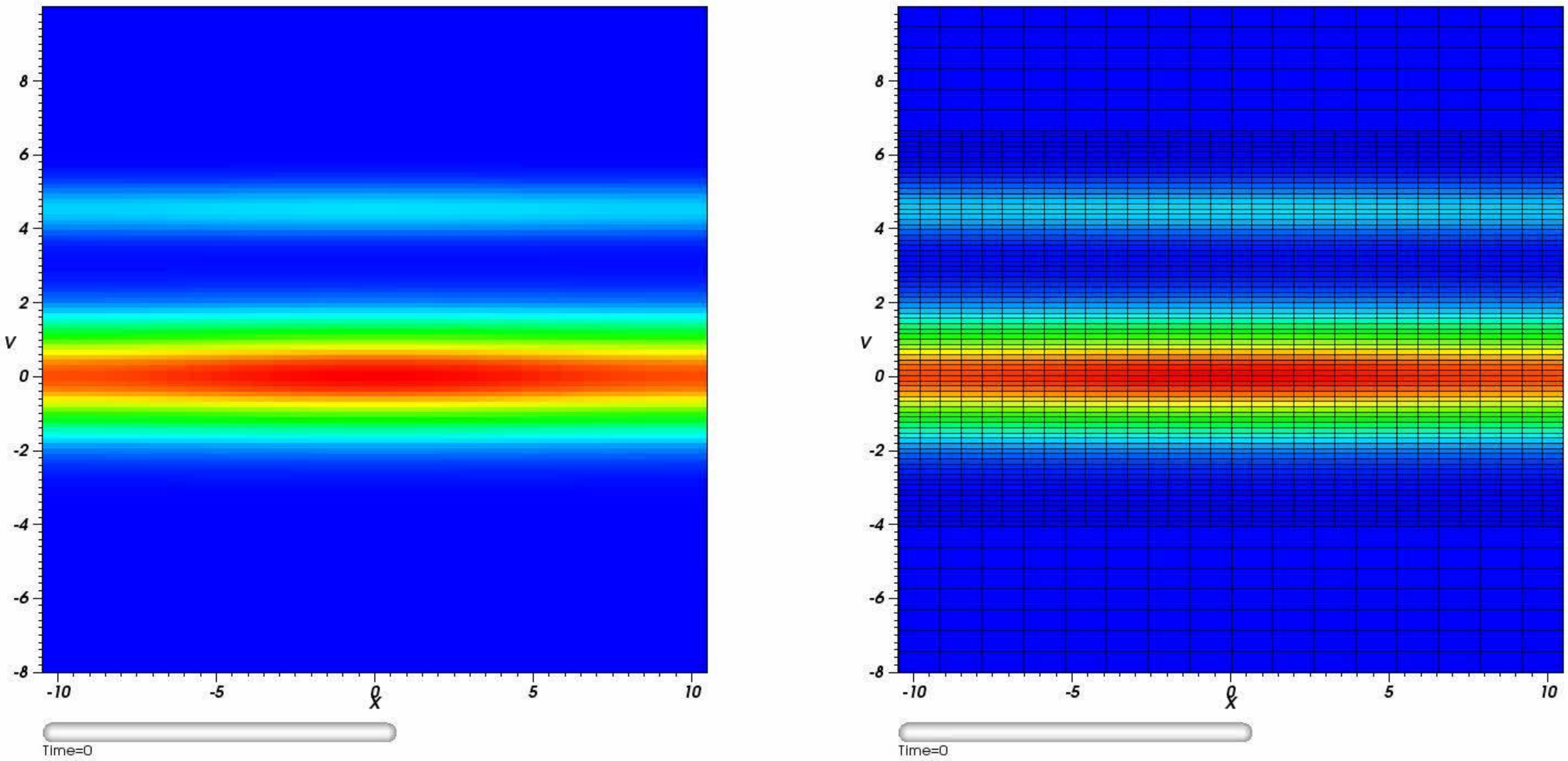
- Results from nonlinear negative frequency shift due to trapped electrons in finite EPWs
- Phase velocity is faster off-axis leading to ***positive wavefront curvature***
- Dominates ***at all times for large amplitude waves*** and ***at later times for small amplitude waves***

Consistent with SRS results of Yin *et al*, *Phys. Rev. Lett.* 99, 265004 (2007), which are in highly nonlinear regime

Reduction algorithm required some work to scale with the configuration space dimension



Distribution function for Bump-On-Tail simulation demonstrating AMR based on SAMRAI



$$f_0(x,v) = [(1+0.04\cos(0.3x)]f_b(v)$$

$$f_b(v) = \frac{0.9}{\sqrt{2\pi}}\exp\left[-\frac{v^2}{2}\right] + \frac{0.2}{\sqrt{2\pi}}\exp\left[-4(v-4.5)^2\right]$$

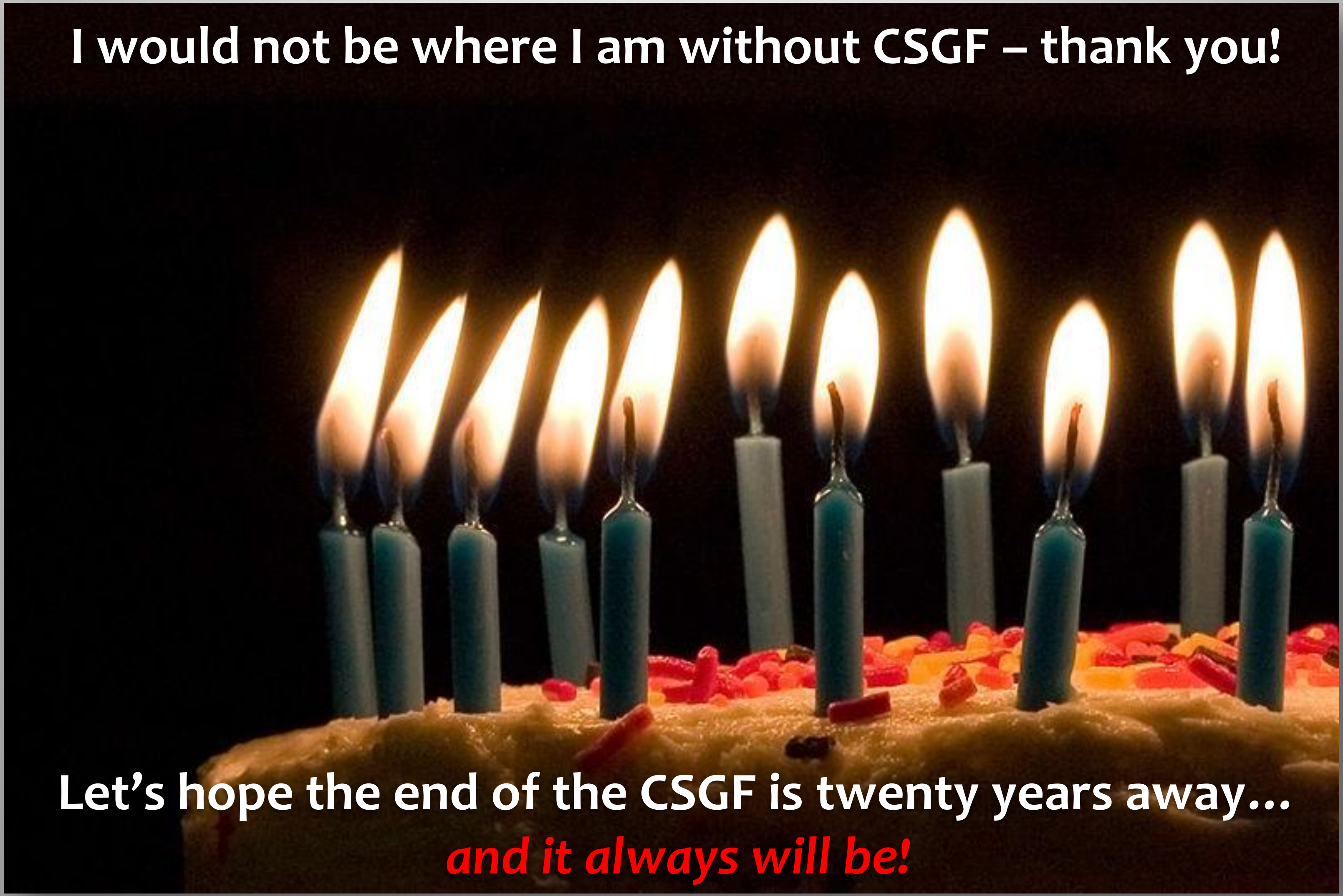
Guide your team but give them freedom to succeed in unanticipated ways

- Passion and a plan sell your ideas and attract a great team
- Understand how to motivate different types of people
- Focus on your goals and be proactive
- Trust your experts' opinions

"If your actions inspire others to dream more, learn more, do more and become more, you are a leader." – John Quincy Adams

Congratulations to CSGF at twenty years!

I would not be where I am without CSGF – thank you!



Let's hope the end of the CSGF is twenty years away…

and it always will be!

Photo: Joey Gannon